2023

MATHEMATICS — HONOURS

Paper : CC-11

(Probability and Statistics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative with proper justification (wherever applicable) : 2×10

(a) If $P(B)=\frac{1}{4}, P(A|B)=\frac{1}{2}, P(B|A)=\frac{1}{4}$, then $P(A)$ is

(i) $\frac{1}{4}$ (ii) $\frac{1}{2}$

(iii) $\frac{2}{3}$ (iv) $\frac{1}{8}$.

(b) Two dice are thrown. Then P (sum of faces = 10) is

(i) $\frac{1}{12}$ (ii) $\frac{1}{3}$

(iii) $\frac{1}{36}$ (iv) $\frac{1}{18}$.

(c) The probability density function is given by

$$f(x)=\begin{cases} x(1-x), & \text{when } -1<x<1 \\ 0, & \text{elsewhere} \end{cases}.$$

Then the value of $P(X>\frac{1}{2})$ is

(i) $\frac{1}{12}$ (ii) $\frac{2}{3}$

(iii) $\frac{5}{12}$ (iv) $\frac{1}{6}$.

(d) If $\phi(t)$ is the characteristic function of a random variable X, then the characteristic function for $\left(\frac{X}{2}-2\right)$ will be

(i) $e^{-2it}\phi(t/2)$ (ii) $e^{-it}\phi(2t)$

(iii) $e^{it/2}\phi(2t)$ (iv) $e^{2it}\phi(t)$.

(e) If the joint p.d.f. of (X, Y) is given by

$$f(x,y)=\begin{cases}\frac{1}{4}, & 0\le x,y\le 1\\ 0, & \text{otherwise}\end{cases},$$

then $P(X+Y\le 1)$ is equal to

(i) $\frac{1}{3}$ (ii) $\frac{1}{16}$

(iii) $\frac{1}{8}$ (iv) $\frac{1}{4}$.

(f) If the moment generating function of a binomial random variable is $M(t)=\left(\frac{2}{3}+\frac{1}{3}e^{t}\right)^{5}$, then the mean and variance are respectively

(i) $\frac{10}{9},\frac{5}{3}$ (ii) $\frac{2}{9},\frac{1}{3}$

(iii) $\frac{1}{3},5$ (iv) $\frac{5}{3},\frac{10}{9}$.

(g) If independent random variables X_1, X_2, X_3 have variance L, then the variance of the variate $\frac{2X_1+4X_2+5X_3}{3}$ is

(i) $2L$ (ii) $5L$

(iii) $3L$ (iv) $10L$.

(h) Let α be a population parameter which is estimated with two statistics A and B and $P(A<\alpha<\beta)=\beta$. Then the confidence coefficient for the interval estimate of α with confidence limits A and B is

(i) β (ii) $\beta-1$

(iii) $1-\beta$ (iv) None of these.

(i) Maximum Likelihood Estimator of α in the distribution function $\alpha e^{-\alpha x}$ will be

(i) $\frac{1}{x}$ (ii) $\frac{1}{x^2}$

(iii) $\frac{1}{\bar{x}}$ (iv) $\bar{x}$.

(j) For $f(x,\theta)=\frac{1}{\theta}; 0 \le x \le \theta$, to test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ in the critical region $1 \le x \le 1.5$, the Type-I error is

(i) 0.5 (ii) 0.75

(iii) 0.25 (iv) 0.15.

Unit - 1

Answer ***any two*** questions.

2. From an urn containing 3 white and 5 black balls, 4 balls are transferred into an empty urn. From the second urn 2 balls are drawn and they happen to be white. What is the probability that the third ball drawn from the same urn will be white? 5

3. A point P is taken at random on a line segment AB of length $2a$. Find the probability that the area of the rectangle $AP.PB$ will exceed $\frac{a^2}{2}$. 5

4. If m and μ_r denote the mean and central r^{th} moment of a Poisson-distribution, prove that $\mu_{r+1} = rm\,\mu_{r+1} + m\frac{d\mu_r}{dm}$. 5

Unit - 2

Answer ***any two*** questions.

5. Let X and Y be continuous random variables with joint probability density function

$$f_{X,Y}(x,y)=\begin{cases}6e^{-(2x+3y)}, & x,y \ge 0\\ 0, & \text{otherwise}\end{cases}.$$

Find $E(Y|X>2)$ and $P(X>Y)$. Are X and Y independent? 4+1

6. If the regression lines are $x+6y=6$ and $3x+2y=10$, find the means and correlation coefficient. 5

7. If $f(x,y)=3x^2-8xy+6y^2$ $(0<x<1,\ 0<y<1)$ find $f_x(x|y)$ and $f_y(y|x)$ and show that X and Y are dependent. 5

Unit - 3

Answer *any one* question.

8. A random variable X has probability density function $f(x)=12x^2(1-x);\ 0<x<1$. Compute $P(|X-m|\geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality where m is the mean and σ is the standard deviation of the distribution. 5

9. By applying central limit theorem to a sequence of random variables with Poisson distribution prove that

$$\lim_{n\to\infty} e^{-n}\sum_{r=0}^{n}\frac{n^r}{r!}=\frac{1}{2}$$ 5

Unit - 4

Answer *any two* questions.

10. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n taken from a normal population with mean zero and standard deviation σ. Show that $\sum_{i=1}^{n}\frac{x_i^2}{n}$ is an unbiased estimate of σ^2. 5

11. Obtain the maximum likelihood estimates of α and β for the random sample $x_1, x_2, \ldots, x_n$ drawn from an exponential population with probability function $f(x,\alpha,\beta)=y_0e^{-\beta(x-\alpha)}, \beta>0,\ y_0$ is a constant $\alpha\leq x\leq\infty$. 5

12. A sample of size 150 is drawn from a population with standard deviation 15. If the sample mean is 40, find the 95% confidence interval of this population mean $\left[\text{Given}: \frac{1}{\sqrt{2\pi}}\int_{1.96}^{\infty} e^{-\frac{x^2}{2}}dx=0.025\right]$. 5

13. Find the sampling distribution of the mean of Poisson (μ) population. ($\mu>0$) 5

Unit - 5

Answer *any two* questions.

14. A die is thrown 60 times with the following results :

Face:	1	2	3	4	5	6
Frequency:	6	10	8	13	11	12

Are the data consistent with the hypothesis that the die is unbiased? 5

[Given : $\chi^2_{0.01}=15.09$ for 5 d.f.]

15. A die was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the die was unbiased? $\left[\text{Given}: \frac{1}{\sqrt{2\pi}} \int_{-2.58}^{2.58} e^{-\frac{x^2}{dx^2}} = 0.95\right]$. 5

16. Given the probability density function $f(x,\theta) = \theta e^{-\theta x}, 0 \le x < \infty, \theta > 0$. The null hypothesis $H_0 : \theta = 2$ against the one sided alternative $H_1 : \theta > 2$ will be tested on the following test procedure : H_0 should be rejected if sample drawn from the population of X is greater than or equal to 6. Find Type-I and Type-II error and power of the test. 5

17. Fit a suitable straight line to the following bivariate data : 5

X	1	1.5	10	20
Y	1.1	1.8	11	20
